P 170,

[2] $M\left(\frac{2+4}{2}, \frac{3+5}{2}\right) = M(3,4)$

$m_{AB} = \frac{5-3}{4-2} = 1$

$m = -1$

$\ell: \quad y-4 = -(x-3)$

$\boxed{y = -x+7}$

[3] $\begin{matrix} \ell_1 \\ \ell_2 \end{matrix} \begin{bmatrix} 3x+4y=18 \\ x-2y=-4 \end{bmatrix} \equiv \begin{bmatrix} 3x+4y=18 \\ 3x-6y=-12 \end{bmatrix} \equiv \quad 10y = 30 \Rightarrow \boxed{y=3}$

$x-6=-4$

$\boxed{x=2}$

Then $y = kx$

$3 = 2k$

$\boxed{k = \frac{3}{2}}$

[4] $(a, a+4), (-2,6), (7,5)$

$m = \frac{6-a-4}{-2-a} = \frac{2-a}{-2-a} = \frac{a-2}{a+2}$

$m = \frac{5-6}{7+2} = \frac{-1}{9}$

$\frac{a-2}{a+2} = \frac{-1}{9}$

$9a-18 = -a-2$

$10a = 16$

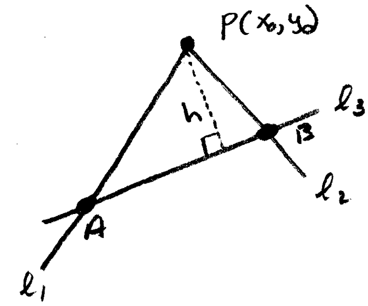
$\boxed{a = \frac{8}{5}}$

P 170, ctd

[5] $\ell_1 \quad x + y = 7$

$\ell_2 \quad 3x - 2y = 1$

$\ell_3 \quad x - 4y + 3 = 0$



Intersection ℓ_1, ℓ_2

$$\begin{bmatrix} x + y = 7 \\ 3x - 2y = 1 \end{bmatrix} \equiv \begin{bmatrix} 2x + 2y = 14 \\ 3x - 2y = 1 \end{bmatrix} \Rightarrow 5x = 15 \Rightarrow \boxed{x_0 = 3}$$

$$3 + y = 7$$

$$\boxed{y_0 = 4}$$

$$h = \frac{|3(1) - (4)(4) + 3|}{\sqrt{1^2 + 4^2}}$$

$$= \frac{|-10|}{\sqrt{17}}$$

$$= \frac{10}{\sqrt{17}}$$

$$h = \frac{10\sqrt{17}}{17}$$

Then

$$A = \tfrac{1}{2} b h$$

$$b = AB$$

$$A : \begin{bmatrix} x + y = 7 \\ x - 4y = -3 \end{bmatrix} \Rightarrow 5y = 10 \Rightarrow \boxed{y = 2,\ x = 5}$$

$$B : \begin{bmatrix} 3x - 2y = 1 \\ x - 4y = -3 \end{bmatrix} \Rightarrow \begin{bmatrix} 6x - 4y = 2 \\ x - 4y = -3 \end{bmatrix} \Rightarrow 5x = 5 \Rightarrow \boxed{x = 1}$$

$$1 - 4y = -3$$

$$4y = 4 \qquad \boxed{y = 1}$$

$$AB = \sqrt{(5-1)^2 + (2-1)^2} = \sqrt{16 + 1} = \sqrt{17}$$

$$A = \tfrac{1}{2}\left[\sqrt{17}\right]\left[\frac{10\sqrt{17}}{17}\right] = 5$$

$$\therefore \boxed{A = 5 \text{ units}^2}$$

[6]

$$\begin{bmatrix} 8A + 4B + C = -80 \\ 3A - B + C = -10 \\ 6A + 8B + C = -100 \end{bmatrix} \Rightarrow \begin{matrix} A = -6 \\ B = -8 \\ C = 0 \end{matrix}$$

Circle:

$$x^2 + y^2 - 6x - 8y = 0$$

$$(x-3)^2 + (y-4)^2 = 9 + 16$$

$\boxed{\therefore \text{center } (3,4), \text{ radius } 5}$

[7]

midpt T of AB $\left(\frac{4+2}{2}, \frac{5-2}{2}\right) = \left(3, \frac{3}{2}\right)$

M internally divides PT in ratio 2:1.

so,

$$x = \frac{(2)(3) + (1)u}{3}, \quad y = \frac{2\left(\frac{3}{2}\right) + (1)v}{3}$$

$$x = \frac{6+u}{3}, \quad y = \frac{3+v}{3}$$

$$u = 3x - 6, \quad v = 3y - 3$$

$$u^2 + v^2 = 9$$

$$(3x-6)^2 + (3y-3)^2 = 9$$

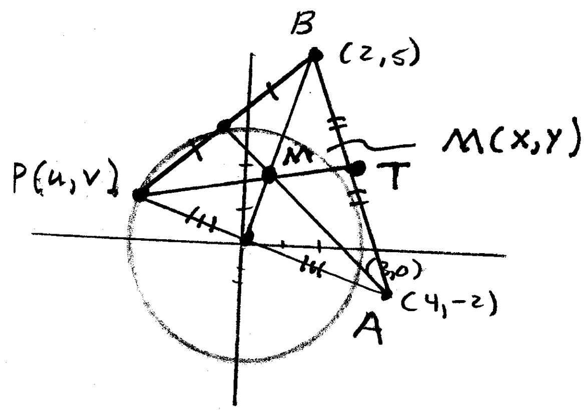
$$9x^2 + 9y^2 - 36x - 18y + 36 + 9 = 9$$

$$x^2 + y^2 - 4x - 2y = -4$$

$$(x-2)^2 + (y-1)^2 = -4 + 4 + 1$$

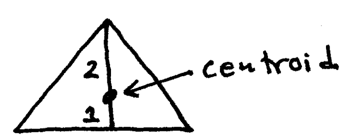
$$(x-2)^2 + (y-1)^2 = 1$$

$\boxed{\therefore \text{Circle Center } (2,1) \text{ radius } 1}$



centroid M is at intersection of three medians of triangle

* centroid is located 2/3 of distance from vertex to opposite side.



* Is a very useful fact that you should remember.

[8] (1, 5) on line tangent to circle.

circle $x^2 + y^2 = 1$

line $x_0 x + y_0 y = 1$, where (x_0, y_0) point of tangency

since x_0, y_0 on circle and (1, 5) on line,

$$\begin{bmatrix} x_0^2 + y_0^2 = 1 \\ x_0 + 5y_0 = 1 \end{bmatrix}$$

$$(1 - 5y_0)^2 + y_0^2 = 1$$

$$26y_0^2 - 10y_0 = 0$$

$$y_0(13y_0 - 5) = 0$$

$y_0 = 0$ <u>OR</u> $y_0 = \frac{5}{13}$

$x_0 = 1$

$$x_0 + \frac{25}{13} = 1$$

$$x_0 = 1 - \frac{25}{13}$$

$$x_0 = \frac{-12}{13}$$

$$-\frac{12}{13}x + \frac{5}{13}y = 1$$

$$-12x + 5y = 13$$

$$12x - 5y = -13$$

Lines are

$$\boxed{\begin{array}{c} x = 1 \\ 12x - 5y = -13 \end{array}}$$